measurement accuracy performed. The high measurement accuracy obtained with these transitions allows coplanar probes to be used to test microstrip circuits without via holes at the wafer level. The need for mounting circuits in fixtures for testing is eliminated, resulting in lower testing costs.

The transition may prove especially useful at millimeter wavelengths, where its size can be reduced. Work to characterize this transition at millimeter wavelengths is in progress. Work to determine the S parameters of the transition as a function of substrate thickness is also in progress.

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Pulse Dispersion Distortion in Open and Shielded Microstrips Using the Spectral-Domain Method

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Abstract —**The spectral-domain method is used to compute the effective dielectric constant [$\epsilon_{r_{\text{eff}}}(f)$] of open and shielded microstrip lines to analyze the dispersion distortion of short electrical pulses. Precise expressions for the longitudinal and transverse current distributions allow a high level of accuracy for $\epsilon_{r_{\text{eff}}}(f)$. It is determined that computation time can be minimized for the open microstrip calculations by using the shielded microstrip formulation provided large dimensions for the conducting walls are taken.**

I. Introduction

The analysis of the transient signal response in microstrip transmission lines is important in microwave integrated circuits (MIC's) when large-bandwidth signals or high switching speeds are considered. Electrical pulses generated from optoelectronic switching typically have wide spectra that extend into the dispersive frequency region of microstrip lines. In the past, transient signal behavior in microstrip lines was analyzed with quasi-static formulas for the effective dielectric constant, $\epsilon_{r_{\text{eff}}}(f)$, of the fundamental mode [1]–[5]. However, detailed pulse dispersion has not yet been examined with $\epsilon_{r_{\text{eff}}}(f)$ obtained from rigorous full-wave analyses such as the spectral-domain method [6], [7]. Neither have comparisons been made of distorted pulses using full-wave methods and quasi-static techniques.

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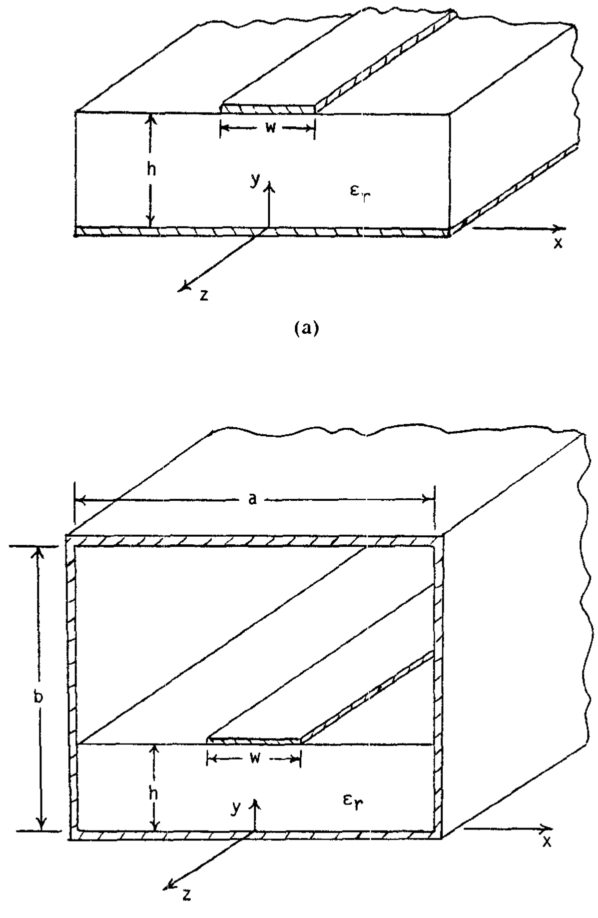

Fig. 1. Configuration and parameters of (a) open microstrip and (b) shielded microstrip lines.

This paper considers the dispersion of short electrical pulses propagating along open and shielded microstrip lines (Fig. 1) with $\epsilon_{r_{\text{eff}}}(f)$ calculated from the spectral-domain method. The accuracy of this method can be increased systematically by including more basis functions for the longitudinal and transverse currents ($J_z(x)$ and $J_x(x)$, respectively). However, as pointed out by Kobayashi, only one basis function for each current component is sufficient if these distributions are very good approximations of the exact currents on the strip conductor [8], [9]. Therefore the expressions for $J_z(x)$ and $J_x(x)$ considered in this paper are accurate formulas which allow the currents to be represented by only one basis function. This corresponds to the "first-order" solution of [6] and [7], while neglecting the transverse current corresponds to the "zero-order" solution. For comparison purposes, the zero- and first-order solutions are used to compute the dispersion curve of open and shielded microstrip lines. Also, the current expressions considered here will be used for both the open and the shielded lines so that their dispersive characteristics may be compared. It is shown that computation time may be minimized by using the shielded line formulation for the open line calculations. Finally, pulses calculated with $\epsilon_{r_{\text{eff}}}(f)$

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from the spectral-domain method, from quasi-static approximations [10], and from curve-fitting methods [11] are compared.

II. Formulation

The main steps of the spectral-domain formulation for both types of microstrips will be reviewed briefly here. The differences in the two formulations will contrast their corresponding computational efficiency in calculating $\epsilon_{r_{\text{eff}}}(f)$.

For the open microstrip case shown in Fig. 1(a), the scalar potentials and the field components derived from them are Fourier transformed with respect to x as

$$\tilde{\psi}_i(\alpha, y) = \int_{-\infty}^{+\infty} \psi_i(x, y)\, e^{+j\alpha x}\, dx \tag{1}$$

with $i = 1,2$ denoting the regions above and below the air–substrate interface, respectively, and the tilde ($\tilde{}$) representing the transformed field quantity. The hybrid-mode fields (TE^z and TM^z) are then modified according to the boundary conditions of the open structure so that the transformed scalar potentials are functions of the longitudinal and transverse currents on the strip conductor.

The solution for β [and $\epsilon_{r_{\text{eff}}}(f)$] involves the utilization of Galerkin's method (essentially a special case of the moment method where the weighting functions are identical to the basis functions) applied in the spectral domain. The basis functions in this case are used to represent the current distributions according to [6]

$$\tilde{J}_x(\alpha) = \sum_{m=1}^{M} c_m \tilde{J}_{xm}(\alpha) \tag{2a}$$

$$\tilde{J}_z(\alpha) = \sum_{m=1}^{N} d_m \tilde{J}_{zm}(\alpha) \tag{2b}$$

where M and N are the number of functions taken to expand $\tilde{J}_x(\alpha)$ and $\tilde{J}_z(\alpha)$, respectively. Applying Galerkin's method with the weighting functions $\tilde{J}_{zi}(\alpha)$ and $\tilde{J}_{xi}(\alpha)$ for different values of i, the result is [6]

$$\sum_{m=1}^{M} K_{im}^{(1,1)} c_m + \sum_{m=1}^{N} K_{im}^{(1,2)} d_m = 0, \qquad i = 1,2,\cdots,N \tag{3a}$$

$$\sum_{m=1}^{M} K_{im}^{(2,1)} c_m + \sum_{m=1}^{N} K_{im}^{(2,2)} d_m = 0, \qquad i = 1,2,\cdots,M \tag{3b}$$

$$K_{im}^{(1,1)} = \int_{-\infty}^{+\infty} \tilde{J}_{zi}(\alpha) G_{11}(\alpha, \beta) \tilde{J}_{xm}(\alpha)\, d\alpha \tag{4a}$$

$$K_{im}^{(1,2)} = \int_{-\infty}^{+\infty} \tilde{J}_{zi}(\alpha) G_{12}(\alpha, \beta) \tilde{J}_{zm}(\alpha)\, d\alpha \tag{4b}$$

$$K_{im}^{(2,1)} = \int_{-\infty}^{+\infty} \tilde{J}_{xi}(\alpha) G_{21}(\alpha, \beta) \tilde{J}_{xm}(\alpha)\, d\alpha \tag{4c}$$

$$K_{im}^{(2,2)} = \int_{-\infty}^{+\infty} \tilde{J}_{xi}(\alpha) G_{22}(\alpha, \beta) \tilde{J}_{zm}(\alpha)\, d\alpha \tag{4d}$$

and $G_{ij}(\alpha, \beta)$ (the elements of the dyadic Green's function in the spectral domain) are given in [6].

The formulation of the shielded microstrip line [7] of Fig. 1(b) is essentially identical to the open microstrip case except that a finite Fourier transform must be used. That is, instead of (1), we have

$$\tilde{\psi}_i(n, y) = \int_{-a/2}^{+a/2} \psi_i(x, y)\, e^{+j\hat{k}_n x}\, dx \tag{5}$$

where $\hat{k}_n = (n - 1/2)2\pi/a$ is taken for the fundamental mode. Using this definition and the same procedure as above with different boundary conditions that reflect the rectangular waveguide geometry, a set of equations identical to (3a) and (3b) are obtained with matrix elements redefined as [7]

$$K_{im}^{(1,1)} = \sum_{n=1}^{\infty} \tilde{J}_{zi}(n) G_{11}(n, \beta) \tilde{J}_{xm}(n) \tag{6a}$$

$$K_{im}^{(1,2)} = \sum_{n=1}^{\infty} \tilde{J}_{zi}(n) G_{12}(n, \beta) \tilde{J}_{zm}(n) \tag{6b}$$

$$K_{im}^{(2,1)} = \sum_{n=1}^{\infty} \tilde{J}_{xi}(n) G_{21}(n, \beta) \tilde{J}_{xm}(n) \tag{6c}$$

$$K_{im}^{(2,2)} = \sum_{n=1}^{\infty} \tilde{J}_{xi}(n) G_{22}(n, \beta) \tilde{J}_{zm}(n) \tag{6d}$$

where $G_{ij}(n, \beta)$ are given in [7]. The discrete summations in (6a)–(6d) converge to a solution for β more rapidly than the integral formulas of (4a)–(4d), as discussed in the next section.

The expression for $J_z(x)$ is critical for the accuracy of $\epsilon_{r_{\text{eff}}}(f)$. The transverse current component, in contrast, merely acts as an adjuster for the magnitude of $\epsilon_{r_{\text{eff}}}(f)$ as determined by $J_z(x)$ [8], [12]. For this investigation, the normalized distribution used for $J_x(x)$ is given by Denlinger [13] (correction given in [9]) as

$$J_x(x) = \begin{cases} \sin\left[\dfrac{\pi x}{0.8w}\right], & 0 \leqslant |x| \leqslant \dfrac{0.8w}{2} \\ \pm\cos\left[\dfrac{\pi x}{0.2w}\right], & \dfrac{0.8w}{2} < |x| \leqslant \dfrac{w}{2} \end{cases} \tag{7}$$

where ($\pm$) ensures continuity at the ($\pm 0.8w/2$) points. The Fourier transform (from the definition of (1)) of (7) is given by

$$\tilde{J}_x(\alpha) = j2w\left[\frac{\alpha w \cos(0.4\alpha w)}{\left(\dfrac{\pi}{0.8}\right)^2 - (\alpha w)^2} + \frac{5\pi \sin(0.5\alpha w) - \alpha w \cos(0.4\alpha w)}{(5\pi)^2 - (\alpha w)^2}\right]. \tag{8}$$

This form of $J_x(x)$ was chosen for its adequate accuracy and its relatively simple Fourier transform expression when compared to that of Kobayashi's expression for the transverse current [8], [9]. Also, calculating $\tilde{J}_x(\alpha)$ given in [8] requires excessive computer time. Therefore, (7) is a good choice when computational efficiency is needed.

For the normalized longitudinal current component, the distribution used here is given by [8], [9]

$$J_z(x) = 1 + 10\left\{1 - \frac{2x_c}{w}\right\}\frac{M(x) - 1}{M(x_c) - 1} \tag{9a}$$

where

$$M(x)=\frac{1}{\sqrt{1-\left(\frac{2x}{w}\right)^2}} \quad (9b)$$

is the normalized Maxwell distribution for an isolated conducting strip [13] and $2x_c/w$ varies with w/h [9, fig. 5]. The Fourier transform of (9a) is [8]

$$\tilde{J}_z(\alpha)=\frac{2}{\alpha}\sin\left(\frac{\alpha w}{2}\right)+\frac{10(1-2x_c/w)}{M(x_c)-1}\cdot\left[\frac{\pi w}{2}J_0\left\{\frac{|\alpha|w}{2}\right\}-\frac{2}{\alpha}\sin\left(\frac{\alpha w}{2}\right)\right] \quad (10)$$

where $J_0(|\alpha|w/2)$ is a Bessel function of zero order. This form of $J_z(x)$ has been shown to be in good agreement with the theoretical longitudinal current as calculated from the Green's function method [9]. For the shielded microstrip line where the definition of (5) is used, the transform variable α in (8) and (10) is replaced with $\hat{k}_n$ for the discrete summations in (6a)–(6d).

The computations for the dispersed waveforms at a distance L along a microstrip line are made from

$$V(t,z=L)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{V}(\omega,z=0)\,e^{j[\omega t-\beta(\omega)L]}\,d\omega \quad (11)$$

where

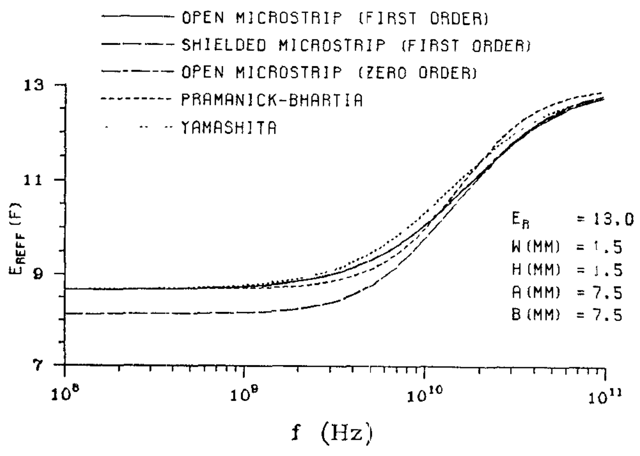
$$\beta(\omega)=\frac{\omega}{c}\sqrt{\epsilon_{r_{\text{eff}}}(\omega)} \quad (11a)$$

and $\tilde{V}(\omega,z=0)$ is the Fourier transform of the pulse at the reference point of the line.

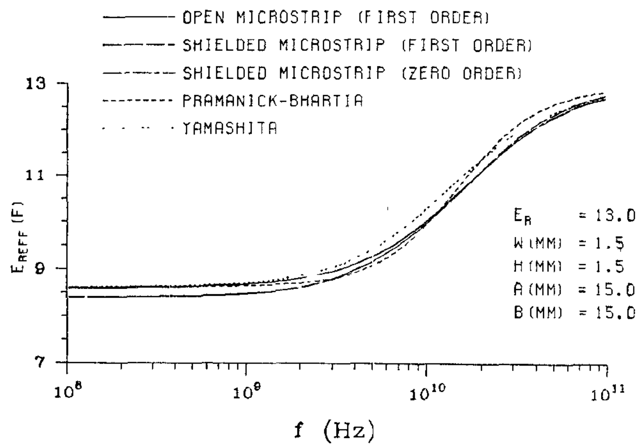
III. Numerical Results and Discussion

The effective dielectric constant (of the fundamental mode) from the spectral-domain method, from the quasi-static formula of Pramanick and Bhartia [10], and from the curve-fitting representation of Yamashita [11] are shown in Fig. 2(a) with $\epsilon_r=13$ for a gallium arsenide substrate. The dimensions of the conducting walls (a and b) are only five times that of the strip width w and substrate height h for this case. Notice in Fig. 2(a) that the shielded microstrip line has significantly lower values for $\epsilon_{r_{\text{eff}}}(f)$ at lower frequencies. This characteristic for the shielded microstrip is caused by the fact that some of the electric field lines from the strip conductor are prematurely terminated at the walls as compared to the open microstrip, where nearly all field lines are terminated at the ground plane. The capacitance of the shielded line is thus decreased as a result of the early termination of the electric field lines. Also notice here that the zero-order solution for the open microstrip is indistinguishable from the first-order solution (the curve for the zero-order solution is hidden by the solid-line curve of the first-order solution). This shows that the transverse current contribution is very small. At high frequencies the $\epsilon_{r_{\text{eff}}}(f)$ values of both lines approach ϵ_r, indicating that the total energy is increasingly confined within the substrate.

When the dimensions a and b are increased to ten times the value of w and h, respectively, as illustrated in Fig. 2(b), the low-frequency values of $\epsilon_{r_{\text{eff}}}(f)$ for the shielded microstrip show an increase and seem to approach those of the open microstrip values. This enlargement of the walls of the confining rectangular waveguide further isolates the center strip and causes fewer field lines to terminate at the walls. Fig. 2(b) also shows that the first-order solution of the shielded microstrip (long-dash line) is nearly indistinguishable from the zero-order solution (long-dash, short-dash line) of the shielded line; this again points out the relative insignificance of the transverse current.



(a)



(b)

Fig 2. Comparison of the effective dielectric constant of open and shielded microstrip lines with (a) $w=h=1.5$ mm, $a=b=5w$, and (b) $w=h=1.5$ mm, $a=b=10w$.

As stated earlier, the shielded microstrip formulation converges to a solution faster than the open line formulation. Using an IBM 3090 for the calculations of β as a function of frequency, it was determined that the computation time for the open microstrip is about three to four times that of the shielded line case. As it is possible to obtain a close approximation of the open microstrip dispersion curve using the shielded microstrip formulas with sufficiently large values of a and b (generally at least ten times larger than the width of the center conductor and height of the substrate), it is advantageous to use the zero-order solution of the latter case for both types of lines whenever the spectral-domain method is utilized.

Fig. 3(a) displays 40-ps-wide (at the half magnitude points) Gaussian pulses after propagating a distance of $L=50$ mm along the line. The dimensions of the microstrip parameters (w, h, a, and b) are identical to those of Fig. 2(a). Note that the pulse propagating along the shielded line is shifted to the left because it travels with a faster phase velocity than the one from the open microstrip. This is a result of the pulse's phase velocity, which is inversely proportional to the square root of $\epsilon_{r_{\text{eff}}}(f)$ ($v_p(f)=c/\sqrt{\epsilon_{r_{\text{eff}}}(f)}$, $c=$ free-space velocity of light). Also note that the pulse from the open microstrip's zero-order solution (which is to

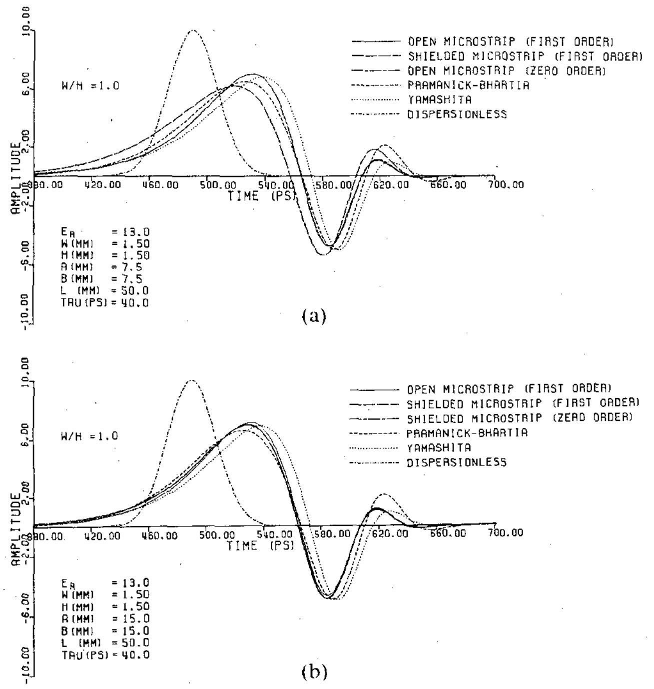


Fig. 3. Comparison of dispersed Gaussian pulses along open and shielded microstrip lines at $L = 50$ mm with (a) $w = h = 1.5$ mm, $a = b = 5w$, and (b) $w = h = 1.5$ mm, $a = b = 10w$.

be represented by the long-dash, short-dash line in Fig. 3(a)) is almost identical to that of its first-order solution (solid line). This obscures the visibility of the zero-order solution. The pulses calculated from the models of Pramanick and Bhartia [10] and Yamashita [11] are fairly close in value to that of the spectral-domain formulation of the open microstrip.

Fig. 3(b) shows the Gaussian pulses with a and b ten times larger than w and h, respectively (same as Fig. 2(b)). Note that pulses from both the open and the shielded microstrip lines (first-order) are in close alignment. The pulse with the zero-order solution of the shielded line cannot be distinguished from the first-order solution. Since the low-frequency values of $\epsilon_{r_{\text{eff}}}(f)$ in Fig. 2(b) are in closer agreement than the case of Fig. 2(a), a corresponding effect in the phase velocities is observed. These results indicate that the shielded line's zero-order solution is able to approximate either the zero- or the first-order solution of the open line.

IV. Conclusions

The spectral-domain method with accurate expressions for the longitudinal and transverse currents was used to calculate the dispersion curve of open and shielded microstrip lines. Dispersion distortions of electrical pulses are computed using $\epsilon_{r_{\text{eff}}}(f)$ from the spectral-domain method and from approximate quasi-static solutions. The values from Pramanick and Bhartia's formula and from Yamashita's formula for the effective dielectric constant were found to be in fairly close agreement of those from the spectral-domain formulation. It was determined that the zero-order solution of the shielded line can be used to obtain either the zero-order or the first-order solution of the open line if sufficiently large values of the shielded line's wall dimensions are taken (at least ten times greater than the center conductor width and substrate height). This gives the advantage of minimizing the computation time for the open microstrip calculations.

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